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# Hybrid Dynamical Modeling of Polycrystalline Shape Memory Alloy Wire Transducers

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## 1. INTRODUCTION

Shape Memory Alloys (SMAs) are a class of smart materials which exhibit different thermo-mechanical properties than conventional metals. When SMA material is heated, transformations in the crystal lattice structure are induced, which generate a macroscopic change in shape on the order of 4-8%. This effect can be exploited for the development of novel actuators that react to an external thermal input with a mechanical deformation (Ballew and Seelecke (2019)). In most applications, SMA material is shaped as a thin wire. In this way, the thermal activation can be simply induced via an electric current, thus resulting in a mechatronic tendon-like actuator.

Despite their remarkable benefits such as compactness, lightweight, and high energy density, SMA materials are characterized by a highly nonlinear response, which is mainly due to a load-, temperature-, and rate-dependent hysteresis. Accurate modeling and compensation of such hysteresis is fundamental for the development of high-performance SMA applications. The goal of this work is to provide an accurate and numerically efficient model, which can be used to perform accurate simulations, model-based design optimization, and control of complex structures driven by polycrystalline SMAs. Our approach is based on a reformulation of the physics-based Müller-Achenbach-Seelecke (MAS) model for polycrystalline SMA wires (Rizzello et al. (2019)) within the hybrid dynamical framework proposed by Goebel et al. (2012). In this way, we are able to significantly reduce the numerical complexity and computation time, without losing numerical accuracy and physical interpretability. In future research, the model will be used for hybrid control of SMA systems.

## 2. SHAPE MEMORY ALLOY MODEL

In the scientific literature, there are plenty mathematical models which describe the behavior of SMA material (Khandelwal and Buravalla (2009)). Due to the different approaches pursued, we can classify them in numerical and analytical/physics-based models. The former are the most computationally efficient ones, but are not suitable for predicting material response to change of external conditions (e.g., load stress, external temperature, structure which is coupled with the SMA). The latter, instead, have a highly sophisticated and detailed description but, in turn, are characterized by strong nonlinearities and require high simulation time.

With the aim to obtaining fast and accurate predictions, in this work we propose a novel physics-based model for one-dimensional polycrystalline SMA wires. To provide meaningful simulation results, the model needs to reproduce as many physical effects as possible. A valuable baseline is offered by the mesoscopic MAS model for polycrystalline SMA material presented by Rizzello et al. (2019). Such a model is based on a novel bookkeping algorithm that reproduces the time evolution of smooth hysteresis loops, as well as inner loops, while maintaining all physical information of the basic single-crystal MAS model (Ballew and Seelecke (2019)). Despite those advantages, the model is affected by slow simulation time due to strong nonlinearities and numerical stiffness of the resulting ODEs.

## 3. HYBRID DYNAMICAL MODEL

Using the work of Rizzello et al. (2019) as starting point, in this section we summarize the theoretical reformulation which permits to overcome the aforementioned numerical limitations. A potential way to improve the physical model implementation consists of eliminating the stiff dynamics of phase transformations, and substitute it with instantaneous hybrid transitions. This approach is similar to what already exploited in a previous work on single-crystal SMA model (Mandolino et al. (2021)), which will now be generalized to more challenging polycrystalline SMAs. The adopted hybrid framework for the SMA model implementation is based on the hybrid theory of Goebel et al. (2012).

A generic stress-strain hysteresis of a superelastic polycrystalline SMA wire is shown in Fig. 1(a). The material produces different pathways (or branches) depending on the entity of a mechanical load applied. By observing this behavior, we can determine three operating modes, each one with well-defined physical interpretations:

(1) **AM**: Austenite to Martensite (or Loading) branch;
(2) **MA**: Martensite to Austenite (or Unloading) branch;

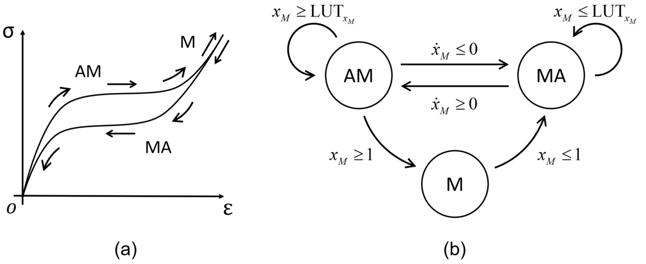


Fig. 1. Example of a qualitative stress-strain hysteresis of polycrystalline SMA wire (a) and the corresponding hybrid automaton with *modes* and *edges* (b).

(3) **M**: Full Martensite branch.

The finite state machine which defines the transition logic between those modes is sketched in Fig. 1(b). A hypothetical operating sequence of the model is as follows. A superelastic SMA wire starts in a full austenitic condition (*mode* **AM**). When subject to an increasing mechanical load, the amount of austenitic crystal lattice reduces while the martensitic one increases. If the load exceeds a certain threshold (dictated by material specifics and external inputs), the SMA wire transforms completely into martensite (*mode* **M**). When the material works below the load threshold and, at the same time, the mechanical load is decreased, the system changes gradually from martensite to austenite (*mode* **MA**). Minor hysteresis loops, which appear when performing partial loading and unloading, are handled by the same **AM** and **MA** *modes*, by considering a novel properly computed unloading branch, see Rizzello et al. (2019) for details. Note that such minor hysteresis loops are not shown in Fig. 1(a), for the ease of clarity.

The finite state machine in Fig. 1(b) can be represented through a generic hybrid system defined as follows:

$$\mathcal{H}\colon \begin{cases} \dot{x} \ = f(x,u) & (x,u) \in C \\ x^{+} \in G(x) & (x,u) \in D \end{cases}. \qquad (1)$$

The states $x := [\varepsilon \ T \ q]^{\intercal} \in \mathbb{X}$, $\mathbb{X} := \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \times \{\mathbf{AM}, \mathbf{MA}, \mathbf{M}\}$, correspond to the SMA strain, temperature, and operative mode. The inputs $u := [v \ J \ T_E]^{\intercal} \in \mathbb{U}$, $\mathbb{U} := \mathbb{R} \times \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0}$, represent wire speed, input power, and environmental temperature, respectively. The output is simply $y := F \in \mathbb{Y}$, $\mathbb{Y} := \mathbb{R}_{\geq 0}$, i.e., the SMA force. Sets $f$ and $C$ describe the continuous-time subset of the system by the respective differential equations and state constraints, while $G$ and $D$ describe the discrete-time subset by difference equations or inclusions and jump constraints. The state-space form of the hybrid SMA model is:

$$\begin{cases} \dot{\varepsilon} \ = v\, l_0^{-1} \\ \dot{T} \ = [J - \lambda A_s(T - T_E) + \dot{L}_{x_M^{(i)}}](\Omega \rho_V c_V - L_T)^{-1} \\ F \ = \pi r_0^2(\varepsilon - \varepsilon_T x_M^{(i)})[E_M^{-1} x_M^{(i)} + E_A^{-1}(1 - x_M^{(i)})]^{-1} \end{cases} \qquad (2)$$

A detailed description of $f$, $G$, $C$, and $D$, as well as variable $x_M^{(i)}$ associated to mode $(i)$, is omitted for conciseness.

## 4. SIMULATION RESULTS

In this section, the behavior of the polycrystalline MAS model from Rizzello et al. (2019) will be compared with the new hybrid one, as well as with experimental stress-strain curves. All simulations are performed in Matlab/Simulink environment. Experimental data and model parameters of a superelastic SMA wire are derived from Rizzello et al. (2019). Due to its nonlinearities, the MAS model requires a stiff solver (i.e., *ode15s*). On the other hand, thanks to its simpler structure, the hybrid reformulation can be also integrated with simpler non-stiff solver (i.e., *ode45*). Different unloading paths are tested, corresponding to two strain rates of $10^{-4}$ $s^{-1}$ and $10^{-2}$ $s^{-1}$, respectively. Simulations results, shown in Fig. 2, demonstrate how the two implementations are practically equivalent in terms of numerical results. An experimental rate-dependency of the hysteresis is also observed, which is well reproduced by both models. The total simulation time of the hybrid model, however, is almost 5.6 times smaller than the one of the MAS, i.e., 36.82 s vs. 206.64 s.

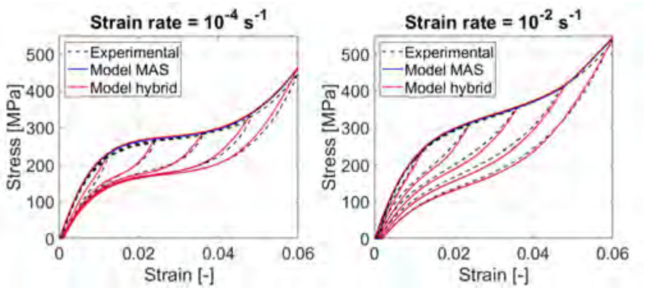


Fig. 2. Hysteresis results for different internal loops at different strain rate: $10^{-4}s^{1}$ (left) and $10^{-2}s^{1}$ (right)

## 5. CONCLUSION

In this paper, a novel physics-based hybrid model for polycrystalline SMA wires is developed and tested. It takes advantage of the hybrid modeling theory to significantly minimize the model complexity, thus allowing to reducing the simulation time without losing accuracy. Future research will focus on using the hybrid polycrystalline model to describe complex SMA systems, as well as on developing hybrid control laws for hysteresis compensation.

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