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# Non-linear RF Device Behavioral Models based on Hammerstein-Wiener Systems

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**Abstract:** Creating behavioral models for radio frequency (RF) devices is a challenging task. Most approaches require a substantial prior knowledge of the physical structure in order to be able to generate suitable mathematical models for the desired characteristics. However, since it is usually not attractive for manufacturers to pass on extensive knowledge about internal components to third parties, one has to rely mainly on black- or gray-box models. An approach is to fit a parameterized model based on representative measurement data, following the example of the Hammerstein-Wiener models. With this approach, only simple linear least squares problems have to be solved and special structures encourage the use of efficient solution methods. In this paper, the general fitting procedure will be discussed and suggestions for successful device modeling will be provided

*Keywords:* Nonlinear system identification, Frequency domain identification, Time series, Gray box modeling, Recursive identification

## 1. INTRODUCTION

In order to integrate various new components from different manufacturers into device simulators, the dynamic behavior of the component under defined operating conditions must be determined as precisely as possible. However, since the physical structure or other internal components are rarely published by the respective semiconductor manufacturers for such purposes, the simulation software usually relies on the models they provide. If these do not exist at all, or only for special simulators, one is faced with a problem. The only remaining option is to create a gray-box model, which is associated with various challenges.

## 2. HAMMERSTEIN-WIENER MODELS

The main challenge one is facing when creating a gray-box model is the choice of an appropriate structure. Here a variety of possible models may apply, ranging from classical RF approaches such as the X-parameter model or even AI models such as neural networks. In this paper, the focus is on a model that is composed of classical elements of signal and system theory, the so-called Hammerstein-Wiener model, and its possible application in the field of RF devices.

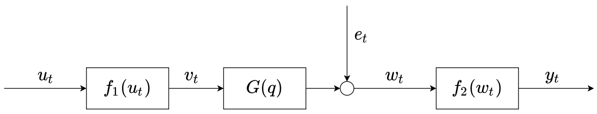


Fig. 1. Hammerstein-Wiener Block Diagram

### *2.1 Structural Overview*

As displayed in Figure 1, a Hammerstein-Wiener model consists of a series connection of an arbitrary non-linear mapping function $f_1(\cdot)$, a discrete LTI system $G(q)$ with the delay operator $q^{-1}$ and another non-linear function $f_2(\cdot)$. We assume that all of these sub-blocks can be described with a set of parameters which may be adapted to fit the input and output measurement data of an actual RF device. Since the Hammerstein-Wiener model is a discrete-time system, these must be available in the form of time domain samples

$$\begin{aligned} Y &= (y_1 \cdots y_N) \\ U &= (u_1 \cdots u_N) \end{aligned} \tag{1}$$

where $N$ is the number of measurement samples. In the following considerations we assume $u_t$, $y_t$ etc. to be a single sample out of a given measurement series. Based on the block diagram in Figure 1, the Hammerstein-Wiener model is given by

$$\begin{aligned} v_t &= f_1(u_t) \\ w_t &= G(q)v_t + e_t \\ y_t &= f_2(w_t) = f_2\left[G(q)f_1(u_t) + e_t\right] \end{aligned} \tag{2}$$

where $f_1(\cdot)$ and $f_2(\cdot)$ are set to be continuous, $f_2(\cdot)$ is furthermore monotone and invertible and $w_t$ is disturbed by a sample $e_t$ of a stationary stochastic process with zero mean. $G(q)$ is an arbitrary transfer function containing the delay operator $q^{-1}$. The nonlinear functions $f_1$ and $f_2$ are approximated with cubic splines as defined in Zhu (2002)

$$\begin{aligned} f_1(u_t) &= \sum_{k=1}^{m_1-2} \alpha_k |u_t - \tilde{u}_k|^3 + \alpha_{m_1-1} + \alpha_{m_1} u_t \\ f_2(w_t) &= \sum_{k=1}^{m_2-2} \beta_k |w_t - \tilde{w}_k|^3 + \beta_{m_2-1} + \beta_{m_2} w_t \end{aligned} \tag{3}$$

where $\tilde{u}_k$ and $\tilde{w}_k$ represent the spline knot sequences. It is established practice to select the spline knots according to the dynamic range of the function argument. However, this formulation cannot be applied as it is, since $w_t$ can not be measured. It is replaced in the course of this section. For the linear time-invariant system $G(q)$ we select a

Box-Jenkins model, which may be replaced with a higher order auto-regressive model with exogenous input (ARX) of order $n$ that reads

$$A(q)w_t = B(q)v_t + e_t \qquad (4)$$
$$w_t = B_1 v_{t-1} + ... + B_n v_{t-n} - A_1 w_{t-1} - ... - A_n w_{t-n}$$

Since the stochastic process $e$ is used to approximate the model error, the objective function $V(\Theta, Z)$ for the upcoming optimization is given by

$$e_t = A(q) f_2^{-1}(y_t) - B(q) f_1(u_t) \qquad (5)$$
$$f_2{}^{-1}(y_t) = \sum_{k=1}^{m_2-2} \gamma_{|} y_t - \tilde{y}_k|^3 + \gamma_{m_2-1} + \gamma_{m_2} y_t$$
$$Z = [u_1, ..., u_N, y_1, ..., y_N], \Theta = [A_1, ..., B_1, ..., \gamma_1, ..., \alpha_1, ...]$$
$$V(\Theta, Z) = \frac{1}{N} \sum_{t=1}^{N} e_t^2$$

where the inverse of $f_2$ is approximated with another spline model with measurable input dynamics for $y_t$ and $\Theta$ contains the parameters of the Hammerstein-Wiener model.

### 2.2 *Optimization Process*

The starting point for the following iterative optimization procedure also defined in Zhu (2002) is based on a small-signal analysis of a non-linear device. With low dynamic ranges, these usually behave almost linearly, which simplifies an initial estimate of the linear ARX model. For this purpose, the following optimization criterion is defined

$$\sum_{t=1}^{N} \left( A_{(0)}(q) f_2{}^{-1}_{(0)}(y_t) - B_{(0)}(q) f_{1(0)}(u_t) \right)^2 \rightarrow \min \qquad (6)$$

where $A_{(0)}(q)$ and $B_{(0)}(q)$ represent the initial ARX coefficients. The non-linear functions $f_1(\cdot)$ and $f_2^{-1}(\cdot)$ are set as identity for this initial estimate. For the subsequent optimization the following steps are repeated until a target norm has been reached. Therefore we are introducing an iteration index $i$ and mark fixed components with the hat-notation

*Step 1:* Determine the spline coefficients $\alpha_k$ for $f_1$ by solving a linear least square problem for fixed $\hat{A}_{(i)}(q)$, $\hat{B}_{(i)}(q)$ and $\hat{f}_{2(i)}^{-1}(y_t)$

$$\sum_{t=1}^{N} \left( \hat{A}_{(i)}(q) \hat{f}_{2(i)}^{-1}(y_t) - \hat{B}_{(i)}(q) f_{1(i+1)}(u_t) \right)^2 \rightarrow \min$$

*Step 2:* Determine the spline coefficients $\gamma_k$ for $f_2^{-1}$ by solving a linear least square problem, where $\hat{A}_{(i)}(q)$, $\hat{B}_{(i)}(q)$ and $\hat{f}_{1(i+1)}[u_t]$ are fixed

$$\sum_{t=1}^{N} \left( \hat{A}_{(i)}(q) f_2{}^{-1}_{(i+1)}(y_t) - \hat{B}_{(i)}(q) \hat{f}_{1(i+1)}(u_t) \right)^2 \rightarrow \min$$

*Step 3:* The last part of the optimization procedure is analogous to the initial ARX step but with the previously computed splines for $f_1(u_t)$ and $f_2^{-1}(y_t)$. Then one solves the least squares problem

$$\sum_{t=1}^{N} \left( A_{(i+1)}(q) \hat{f}_2{}^{-1}_{(i+1)}(y_t) - B_{(i+1)}(q) \hat{f}_{1(i+1)}(u_t) \right)^2 \rightarrow \min$$

### 2.3 *Modifications*

The original idea of this optimization procedure originates from Zhu (2002), but is associated with several limitations there. For example, it is assumed that $u_t$, $y_t$ and the Hammerstein-Wiener model parameters each represent scalar values. It is therefore only possible to model Single-Input Single-Output (SISO) systems with this approach. However, since this assumption is inadequate for RF devices with multiple ports, a notation for modeling Multiple-Input Multiple-Output (MIMO) systems is proposed in the following where $u_t \in \mathbb{R}^{\mathrm{K}}$ and $y_t \in \mathbb{R}^{\mathrm{L}}$. There are various possibilities to modify the given structure of the Hammerstein-Wiener model so that multivariate data can be approximated. One option are multivariate splines, but a simpler solution is a higher-dimensional ARX model in which the coefficients are represented by matrices. $A_1 ... A_n \in \mathbb{R}^{\mathrm{LxK}}$ and $B_1 ... B_n \in \mathbb{R}^{\mathrm{LxK}}$ therefore applies. Then it is sufficient to apply scalar spline functions to every element within $u_t$ and $w_t$ similarly to the scalar case in Section 2.2. Another improvement is the use of b-Splines, where the order can be defined flexibly.

## 3. NUMERICAL TEST

The variants of the Hammerstein-Wiener models presented here have so far proven themselves in practical use, although the proof of convergence is still work in progress. In the course of this work, mainly models from the Cadence AWR Design Environment were used for verification. An example of a frequency-dependent non-linear common emitter amplifier with a two-tone signal at the input port is provided in Figure 2. As expected, the actual measurement data show attenuated artifacts around the harmonics of the input signal (100 kHz), which a Hammerstein-Wiener model ($n = 3$, $m_1 = m_2 = 11$) approximates accurately.

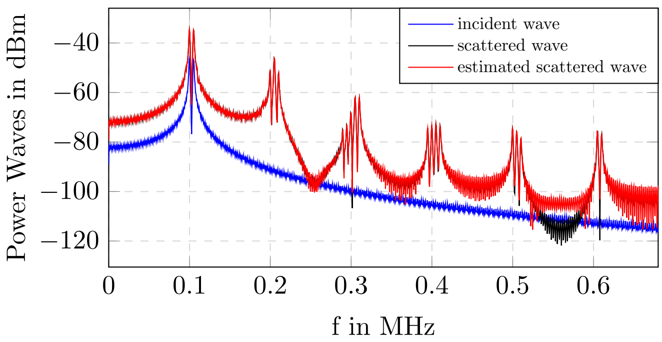


Fig. 2. Scattered Wave Approximation at Port 1

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