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Dynamics of a Mecanum Wheel Pair with Variable Orientation of the Rollers ⋆

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1. INTRODUCTION

The demand of mobile platforms for the motion in a constrained spaces in complex environments or personal high-maneuverability robots for disabled persons has led to the invention of new types of wheels. Beginning from the first patent received by Grabowezky in USA in 1919, engineers started developing wheels that could move not only in their own plane but, for example, perpendicularly to this plane. A key issue for the effective usage of these wheels and for the optimal control of the entire mobile system is the understanding of the physical interaction between the wheels and the environment. For this reason, the mechanics of motion with such wheels draws attention of both researchers and engineers (Campion et al. (1996), Ostrowski and Burdick (1998) and others). The motion of a platform with four Mecanum wheels is investigated in Zeidis and Zimmermann (2019) within the framework of non-holonomic mechanics. In this paper the dynamic equations of a wheel pair that contain two Mecanum wheels with controllable orientation of the rollers during motion is considered. Such a system consists of two co-axial disks with rollers attached to them as shown in Fig. 3. When one of the disks turns with respect to the other disk, the angle of inclination of the rollers relative to the wheel's plane changes. The relative rotation of the disks can be produced by a separate actuator, which allows choosing an optimal orientation of the rollers for a given trajectory.

2. MATHEMATICAL PROBLEM

The Mecanum wheel pair moves so that all its wheels have permanent contact with the underlying plane. The distance between the centers of the wheels is $2l$. The coordinates of the center of mass C in a fixed coordinate system XOY are x_c, y_c , the angle formed by the axis that is orthogonal to the axis of wheel pair with axis OX we denote by ψ. The angles of rotation of the wheels relative to the axes that are perpendicular to the planes of the respective wheels and pass through their centers are φ_i, and the time-dependent torques applied to the wheels are $M_i(t)$ $(i = 1, 2)$.

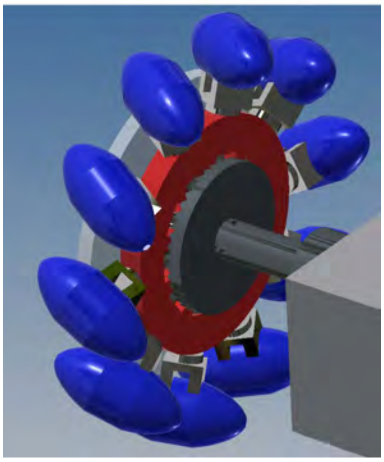

Fig. 1. Prototype of a Mecanum wheel with controllable angle of the rollers

A Mecanum wheel is a wheel with rollers fixed on its outer rim. The axis of each of the rollers forms the same angle δ $(0° \le \delta < 90°)$ with the plane of the wheel. Each roller may rotate freely about its axis, while the wheel may roll on the roller. We will model a Mecanum wheel by a thin disk of radius R, see Fig. 2. Let $\boldsymbol{V}_K$ be the velocity of the wheel's center K, $\boldsymbol{\gamma}$ be the unit vector of the roller's axis, and φ be the angle of rotation of the wheel about the axis that is perpendicular to the wheel's plane and passes through its center. The wheels move without slip, which implies the constraint (Nejmark and Fufaev (1972))

$$\boldsymbol{V}_K \cdot \boldsymbol{\gamma} = R\,\dot{\varphi}\cos\delta\,. \tag{1}$$

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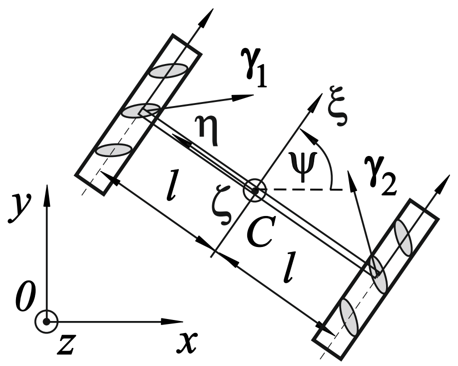

Fig. 2. Mecanum wheel pair

We introduce a wheel pair-attached coordinate system $\xi\eta\zeta$ with origin at the center of mass C of the axis between the wheel pair (Fig. 2). We point axis $C\xi$ orthogonal to the axis of the wheel pair, axis $C\eta$ along the axis of the wheel pair, and axis $C\zeta$ vertically upward. Denote by $V_{C\xi}$ and $V_{C\eta}$ the projections of the velocity of the center of mass onto the movable axes $C\xi$ and $C\eta$, respectively, and represent expression (1) as follows:

$$V_{C\xi}\cos\delta_1 - V_{C\eta}\sin\delta_1 - l\,\cos\delta_1\,\dot{\psi} = R\,\cos\delta_1\,\dot{\varphi}_1\,,$$
$$V_{C\xi}\cos\delta_2 + V_{C\eta}\sin\delta_2 + l\,\cos\delta_2\,\dot{\psi} = R\,\cos\delta_2\,\dot{\varphi}_2\,. \qquad (2)$$

Here the angles $\delta_1 = \delta_1(t)$, $\delta_2 = \delta_2(t)$ are given functions of time t. Then the components $\dot{x}_c$ and $\dot{y}_c$ of the velocity vector of the center of mass in the fixed reference frame are as follows:

$$\dot{x}_c = V_{C\xi}\cos\psi - V_{C\eta}\sin\psi,$$
$$\dot{y}_c = V_{C\xi}\sin\psi + V_{C\eta}\cos\psi. \qquad (3)$$

The configuration of the mechanical system is defined by five generalized coordinates, $q_1 = \varphi_1$, $q_2 = \varphi_2$, $q_3 = \psi$, $q_4 = x_c$, and $q_5 = y_c$. Two generalized velocities can be expressed in terms of the remaining generalized velocities by using the non-holonomic constraint equations (3). The coefficients in these equations be functions of only the independent coordinates and time t. Chaplygin's systems (Papastavridis (2002), Zimmermann et al. (2009)) are usually defined as mechanical systems with non-holonomic time-invariant constraints that are linear with respect to the generalized velocities and can be reduced to the form in which the dependent generalized velocities are expressed in terms of the independent generalized velocities in such a way that the coefficients of the independent generalized velocities are functions only of the independent generalized coordinates. In this case, the dynamic equations can be represented in a special form that are called Chaplygin's equations. Chaplygin's equations form a closed system that does not involve the constraint equations, as it is the case for systems with holonomic constraints. This remarkable property remains valid for the systems with linear time-varying constraints if the coefficients of the independent generalized velocities depend only on the independent generalized coordinates and on the time.

Fig. 3 depicts the time histories $x_c(t)$ and $y_c(t)$ of the coordinates of the wheel pair center of mass for the case where the rollers inclination angle changes periodically with a period of 2 s first from 5° to 85° and then from 85° to 5° (curves 1) and for the case where the roller inclination angle is constant and is equal to 45° (curves 2).

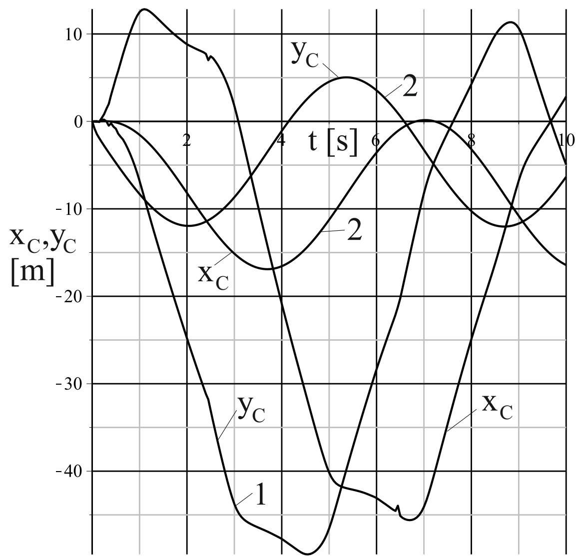

Fig. 3. The time histories of the x_c- and y_c-coordinates of the center of mass of the wheel pair for the cases of changing and constant inclination angle of the rollers

3. CONCLUSION

The equations of motion of a wheel pair with time varying change in the angle of inclination of rollers to the wheel's plane are presented. These equations can be regarded as a modification of Chaplygin's equations for non-holonomic systems with time-varying constraints. The modified equations contain additional terms as compared with the classical equations for the systems with time invariant constraints. The main property of Chaplygin's equations is that the dynamic equations can be integrated irrespective of the constraint equations. This property is retained for the modified equations. The ability of a controllable inclination angle of the rollers of the Mecanum wheels during motion enhances the kinematic possibilities of mobile robots with such wheel pairs.

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