MATHMOD 2022 Discussion Contribution Volume, 10th Vienna Conference on Mathematical Modelling, Vienna, Austria, July 27-29, 2022

Modeling and Control of a Two-body Limbless Crawler on a Rough Inclined Plane ⋆

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1. INTRODUCTION

A mobile system consisting of two interacting bodies regarded as point masses is a simplest model of a worm-like limbless crawler. The interaction force plays the role of a control variable. This system can move progressively on a rough plane with Coulomb's friction acting between each of the bodies and the plane. On a horizontal plane, if both bodies did not move at the initial time instant, this system can move only along the line that connected the bodies at the initial time instant. This is the case, because no lateral impressed forces act on the system. The motion of a two-body crawling system along a straight line on a horizontal plane was studied by Chernousko (2002) and Chernousko (2011). The situation changes for an inclined plane, since the gravity force has a projection onto the direction orthogonal to the line that connects the bodies, provided that both bodies do not lie on the common line of maximum slope. The motion of a two-body crawler on an inclined plane along a line of maximum slope is addressed by Figurina (2018). The aim of this study is to show that the two-body crawler can, in principle, be driven from any initial state of rest to an arbitrarily small neighborhood of any terminal state of rest on an inclined plane, if at the initial time instant the bodies do not lie on the common line of maximal slope.

2. STATEMENT OF THE PROBLEM

Consider a system of two interacting point bodies of masses m and M, $m < M$, on a rough inclined plane. Let k be the coefficients of Coulomb's friction between the bodies and the plane, γ the inclination angle of the plane $(0 < \gamma < \pi/2)$, $\mathbf{F}$ the interaction force applied by body M to body m. We assume that from a state in which both bodies lie on the common line of maximum slope and do not move, body m can be moved upward along this line, with body M remaining at rest. This assumption implies the inequality

$$\tan\gamma \le k\frac{M-m}{M+m}. \qquad (1)$$

⋆ This study was partially supported by the Ministry of Science and Higher Education of the Russian Federation within the framework of the Russian State Assignment under contract No. AAAA-A20-120011690138-6 and partially supported by RFBR Grant No. 20-01-00378.

Let at the initial time instant both bodies be not moving and the bodies do not lie on the common line of maximum slope. The issue we are interested in is whether the system can be transferred into an arbitrary terminal state of rest on the plane. We investigate this possibility in principle and do not impose constraints on the control force and the relative displacement of the bodies. In particular, the impulsive interaction force $\mathbf{F}$ that changes instantaneously the distance between the bodies is allowed and, moreover, the bodies are allowed to pass through each other. It will be shown that such a transfer can be performed by combining infinitely slow (quasistatic) motions of body m and fast motions in which the distance between the bodies changes virtually instantaneously.

3. QUASISTATIC MOTIONS

Consider the quasistatic motion of the system, i.e., the slow motions that can be regarded as a continuous sequence of equilibria. As follows from inequality (1), in the quasistatic mode, only body m can move and body M remains at rest. Introduce in the inclined plane the coordinate system Mxy (fixed for the case of the quasistatic motion), the y-axis of which points upward along the line of maximum slope (Fig.1).

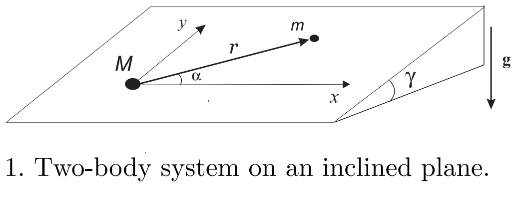

Fig. 1. Two-body system on an inclined plane.

Let r and α denote the polar coordinates of body m in the inclined plane, related to the pole M and the polar axis Mx. The trajectories of the quasistatic motion of body m for $\alpha \in (-\pi/2, \pi/2)$ are defined by the equation

$$\frac{dr}{d\alpha} = \pm\frac{r\sqrt{1-a^2\cos^2\alpha}}{a\cos\alpha}, \quad a = \frac{\tan\gamma}{k} < 1, \qquad (2)$$

The minus sign on the right-hand side of Eq. (2) corresponds to the repulsive motion for which the force $\mathbf{F}$, applied to body m is directed along the vector $\overrightarrow{Mm}$, and the plus sign corresponds to the attractive motion.

Let $r_{\pm}(\alpha, \alpha_0, r_0)$ denote the solution of Eq. (2) subject to the initial conditions $r(\alpha_0) = r_0$. The function r_+ (r_-) monotonically increases (decreases) as α increases in the interval $(-\pi/2, \pi/2)$. The function r_+ possesses the following properties:

$$\lim_{\alpha\to\pi/2} r_+(\alpha)\cos\alpha = \infty, \qquad \lim_{\alpha\to-\pi/2} r_+(\alpha) = 0.$$

The trajectories r_- are symmetric to r_+ about the axis Mx, i.e.,

$$r_-(\alpha, \alpha_0, r_0) = r_+(-\alpha, -\alpha_0, r_0).$$

The quasistatic trajectories of body m are plotted in Fig. 2. It can be shown that body m can move quasistatically from the point (α_0, r_0) clockwise along a curve that is arbitrarily close to the circumferential arc $r = r_0$, $\alpha \in (-\pi/2, \alpha_0]$. We will call such a motion quasistatic circumferential motions (motions along a circumference). The circumferential motions require infinitely frequent switchings between the attractive and repulsive trajectories.

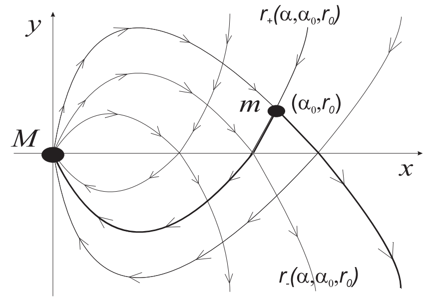

Fig. 2. Quasistatic trajectories of body m.

For $\alpha \in (\pi/2, 3\pi/2)$, the repulsive and attractive trajectories are symmetric with respect to the axis My to the respective trajectories for $\alpha \in (-\pi/2, \pi/2)$. For $\alpha \in [\alpha_0, 3\pi/2)$, body m can move quasistatically along a circumference counterclockwise.

4. FAST MOTIONS. ALGORITHMS FOR DRIVING THE SYSTEM INTO THE TERMINAL STATE

By fast motions we understand the motions that transfer the system between two states of rest in an infinitesimal time. For such motions, the interaction force of the bodies is much larger than the force of friction and, therefore, the center of mass of the system and the line that connects the bodies remain fixed. By assumption, the bodies may pass through each other and, hence, by means of a fast motion body M can be driven to any position on the initial line Mm; then the position of body m is defined uniquely.

By combining fast and quasistatic motions one can drive body m into any position on the plane, with body M remaining arbitrarily close to its initial position. We will prove this proposition for the particular case where $\alpha \in (\pi/2, 3\pi/2)$ for the initial position and $\alpha \in (-\pi/2, \pi/2)$ for the terminal position. The respective process is illustrated in Fig. 3. The starting position of body m is denoted by A and the destination position by B. The larger and smaller circles depict the successive positions of bodies M and m, respectively. The shading density of the circles decreases as the later positions are depicted. First, we proximate body m quasistatically to body M by a distance of $r = \varepsilon$. If ε is small enough, the angle α is close to $3\pi/2$. The respective position of body m corresponds to point C. After this, we perform a fast motion as a result of which bodies m and M change their positions to the positions that are symmetric to the previous positions with respect to the system's center of mass, body m comes into point D, while the change in the position of body M is less than ε. Then, body m is moved quasistatically along a circumference clockwise until it arrives at the point E that belongs to the repulsive trajectory that passes through the desired point B. Finally, body m is moved quasistatically along this repulsive trajectory into the point B. Simplifying, we can regard the described strategy (for $\varepsilon \to 0$) as proximation of bodies m and M until coincidence, with following motion of body m along an appropriate quasistatic repulsive trajectory emerging from the origin.

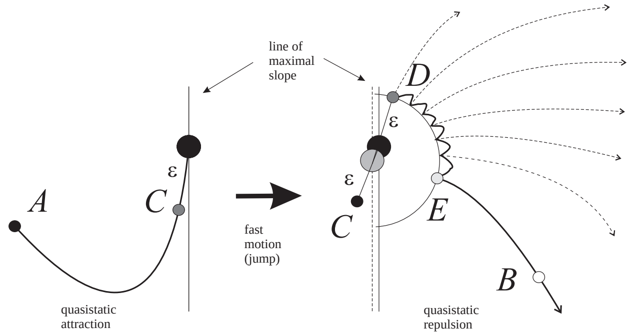

Fig. 3. Motion of body m between two prescribed points, with body M remaining close to its initial position.

The entire strategy for moving the system into the given terminal state can be briefly described as follows. By alternating fast motions of the system and quasistatic motions of body m along a circumference, we move the system into a position in which the line mM passes through the terminal position of body M. Then, by fast motion, we transfer body M into the terminal position. Finally, by using the strategy that was described above, we move body m into the terminal position, the change in the position of body M being able to be made arbitrarily small.

5. CONCLUSION

If at the initial state of rest the bodies of the crawler do not lie on a common line of maximum slope, the system can be driven into an arbitrarily small neighborhood of any terminal state of rest on an inclined rough plane by combining quasistatic and fast motions.

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