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Periodic Regimes of Motion of Capsule System along Straight Line with Dry Friction ⋆

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1. INTRODUCTION

The capsule system driven by a periodically moving internal mass was considered in a number of papers, see Chernousko (2008); Yan et al. (2017). Different control and optimization problems were solved for such systems. The solution of such problems is always searched among the motions with periodic velocity of the capsule, because such motions provide prolonged positive averaged displacement of the system. But the uniqueness and stability of such periodic regimes of motion is not sufficiently studied. For the case when the medium resistance is a monotonous continuous function of the velocity of the capsule and the velocity of the internal mass relative to the capsule is continuous, it was proved, that the periodic regime of motion exists, is unique, and the velocity of all other motions converge to the periodic one exponentially, see Knyaz'kov and Figurina (2020). The same results were obtained in Figurina and Knyazkov (2022) for a system of several interacting bodies and capsules.

In the current paper, a capsule system with an internal mass moves on a plane with dry friction, and the relative velocity of the mass has discontinuities (jumps). These jumps may occur due to collisions in the system. It is proved, that the periodic regime of motion exists and the velocity of any motion converges to the periodic velocity exponentially or in finite time. In contrary to the results obtained in Knyaz'kov and Figurina (2020); Figurina and Knyazkov (2022) for similar locomotion systems, in the current paper, the periodic by velocity motion may be non-unique. This non-uniqueness appears due to jumps in velocity of the capsule for the case of dry friction between the capsule and the plane.

2. PROBLEM STATEMENT

The capsule of mass M contains an internal body of mass m. The position $l(t)$ of the internal mass relative to the capsule changes periodically as a result of some forces, that are internal for the system:

$$l(t+T) = l(t).$$

The capsule moves with the velocity $v(t)$ along a straight line on a rough plane (see Fig. 1). Dry friction force R acts on the capsule. The equation of motion can be written as

$$\dot{v} = u + r(v,u), \tag{1}$$

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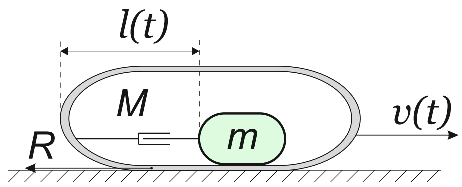

Fig. 1. Scheme of the motion

where r is the normalized dry friction force, $r = \frac{R}{M+m}$, u is the normalized relative acceleration of the internal mass, $u = -\frac{m}{M+m}\ddot{l}$. According to Coulomb's law of dry friction,

$$r(v,u) = \begin{cases} -\mu\,\mathrm{sign}v, & v \neq 0, \\ -u, & v = 0,\ |u| \leq \mu, \\ -\mu\,\mathrm{sign}u, & v = 0,\ |u| > \mu, \end{cases} \tag{2}$$

where $\mu = kg$, k is the coefficient of dry friction, g is the gravitational acceleration. We assume, that for $t \in [0, T]$

$$u(t) = u_0(t) + \sum_{i=1}^{N} d_i \delta(t - t_i),$$

where δ is Dirac delta function, $u_0(t)$ is a periodic piecewise-continuous function. As far as the relative motion of the internal mass is periodic, we have

$$u(t+T) = u(t), \qquad t \geq 0, \tag{3}$$

$$\int_0^T u(t)dt = 0. \tag{4}$$

The details of the statement of the problem can be found in Knyaz'kov and Figurina (2020). We are interested in the existence, uniqueness, and stability of the periodic solution $v_*(t)$ of the problem (1)-(4), such that

$$v_*(t+T) = v_*(t).$$

3. MAIN RESULTS

The following results regarding the periodic solution $v_*(t)$ and the behavior of velocities $v(t)$ of motions with any initial velocity are obtained.

Lemma 1. The distance between any two solutions $v(t)$, $\tilde{v}(t)$ of the equation (1) does not increase:

$$\frac{d}{dt}|v(t) - \tilde{v}(t)| \leq 0.$$

It follows from (1) and (2). From Lemma 1, the following corollary can be easily obtained.

Corollary 1. If there exist two periodic solutions v_*, $\tilde{v}_*$ of the equation (1), they differ by a constant:

$$v_*(t) = \tilde{v}_*(t) + C.$$

Theorem 1. The periodic solution $v_*(t)$ of the problem (1)-(4) exists. If the periodic solution is non-unique, then the set of periodic solutions consists of all the solutions with initial values from an interval $[v_*^{min}(0), v_*^{max}(0)]$.

Proof. The idea of the proof is the following. For the initial values $v^+(0) = \max|u_0(t)|T + \sum_{i=1}^{N} |d_i|$, $v^-(0) = -v^+(0)$, we have $v^+(0) \geq v^+(T)$, $v^-(0) \leq v^-(T)$. Due to continuity, there exists such $v_*(0) \in [v^-(0), v^+(0)]$ that the corresponding solution $v_*(t)$ is periodic.

Let v_*^{min}, v_*^{max} are periodic solutions with minimum (maximum) possible initial values, $v(0) \in (v_*^{min}(0), v_*^{max}(0))$. Due to Lemma 1, the distances $|v_*^{min} - v|$, $|v_*^{max} - v|$ between the solutions do not increase. From Corollary 1, $v_*^{max} = v_*^{min} + C$. Thus, $v = v_*^{min} + \widetilde{C}$, and v is periodic.

The behavior of non-periodic solutions is described by the following theorem.

Theorem 2. Any solution $v(t)$ of the problem (1)-(4) such that $v(0) > v_*^{max}(0)$ converges to the periodic solution $v_*^{max}(t)$. Any solution $v(t)$ of the problem such that $v(0) < v_*^{min}(0)$ converges to the periodic solution $v_*^{min}(t)$.

Proof. Due to Lemma 1, any solution v with $v(0) > v_*^{max}(0)$ tends to $v_*^{max} + C$. It can be shown that $v_*^{max} + C$ is also the solution of (1)-(4). By definition, v_*^{max} is the periodic solution with the maximum possible initial value $v_*^{max}(0)$, hence, $C = 0$ and v converges to v_*^{max}. The second part of the theorem is proved in a similar way.

The following theorem gives a criteria for the type of this convergence.

Theorem 3. (A) Solution $v(t)$ such that $v(0) > v_*^{max}(0)$ converges to the periodic solution $v_*^{max}(t)$ in a finite time if and only if there exists a time instant τ such that $v_*^{max}(\tau) = 0$, and either $v_*^{max}(t) < 0$, $|u(t)| \leq \mu$ or $v_*^{max}(t) \equiv 0$, $-\mu \leq u(t) < \mu$ take place in some left vicinity of the point τ.

(B) Solution $v(t)$ such that $v(0) < v_*^{min}(0)$ converges to the periodic solution $v_*^{min}(t)$ in a finite time if and only if there exists a time instant τ such that $v_*^{min}(\tau) = 0$, and either $v_*^{min}(0) > 0$, $|u(t)| \leq \mu$ or $v_*^{min}(t) \equiv 0$, $-\mu < u(t) \leq \mu$ take place in some left vicinity of the point τ.

(C) If a non-periodic solution $v(t)$ does not converges to the periodic solution $v_*^{max}(t)$ (or $v_*^{min}(t)$) in a finite time, it converges to $v_*^{max}(t)$ (or $v_*^{min}(t)$) exponentially.

Proof. Let v be a non-periodic solution such that $v(0) > v_*^{max}(0)$. If $v_*^{max}(t) \equiv 0$, $-\mu \leq u(t) < \mu$ for $t \in [\tau - a, \tau]$, then a distance between $v(t)$ and $v_*^{max}(t)$ decreases by a constant value over every time period. If $v_*^{max}(t) < 0$, $|u(t)| \leq \mu$ for $t \in [\tau - a, \tau)$, then there exists such time moment $t_0 \in [\tau - a + nT, \tau + nT]$ that $v(t_0) = 0$, $v(t) = 0$, $t \in (t_0, \tau + nT]$, and $v(t) = v_*^{max}(t)$ for all $t \geq \tau + nT$. Thus, part (A) is proved. (B) is proved in a similar way.

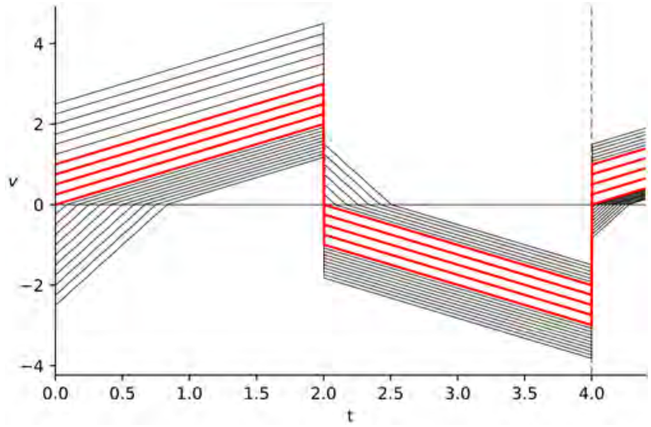

Fig. 2. Example of non-uniqueness of the periodic regime

If a non-periodic v does not converge to v_* in a finite time, there is infinite number of intervals where v and v_* have different signs. From (1), (2), the distance between v and v_* decreases with the rate 2μ on these intervals. It can be proved that the total length of these intervals is sufficiently large, thus, the exponential convergence takes place.

Consider the example, that illustrates the behavior of the velocities $v(t)$ for different initial velocities $v(0)$.

Example. Let's take take $\mu = 1$, $T = 4$, $u(t) = u_0(t) - 3\delta(t - T/2) + 3\delta(t - T)$, where $u_0(t) = 2$ for $t \in [0, T/2)$ and $u_0(t) = -2$ for $t \in [T/2, T)$. The corresponding velocities for different initial values are shown in Fig. 2. The velocities of periodic and non-periodic regimes are shown by red and black colors correspondingly. Here we have $v_*^{min}(0) = 0$, $v_*^{max}(0) = 1$.

If the initial velocity of the capsule $v(0)$ is greater than $v_*^{max}(0)$, then $v(t)$ converges to the motion with $v_*^{max}(t)$, and average velocity of the capsule is directed to the right. If the initial velocity of the capsule $v(0)$ is less than $v_*^{min}(0)$, then $v(t)$ converges to the motion with $v_*^{min}(t)$, and average velocity of the capsule is directed to the left. And there exists some initial velocity $v(0) \in [v_*^{min}(0), v_*^{max}(0)]$, that the capsule returns to its initial state at the end of each time period. This can be used to control vibro-driven capsule robots, because it gives us the ability to influence the direction of movement of the capsule only by specifying its initial velocity. Note, that all periodic solutions $v_*(t)$ with initial velocities $v_*(0) \in [v_*^{min}(0), v_*^{max}(0)]$ are not asymptotically stable.

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