Planar Bipedal Walking Robot with Differentially Flat Dynamics

Akshay Kumar [*] **Vivek Sangwan** [**]

[*] *Indian Institute of Technology Bombay, Mumbai,India (e-mail: akshayk1741@iitb.ac.in).*
[**] *Indian Institute of Technology Bombay, Mumbai,India (e-mail: vivek.sangwan@iitb.ac.in).*

Abstract: Under-actuated bipedal walking robots are an active area of research. Previously, Sangwan and Agrawal (2007) have presented a mechanical design methodology for under-actuated bipeds based on placing center-of-mass of the legs at the hip joint that renders dynamics of a class of planar bipedal walking robots differentially flat. Once this class of under-actuated bipeds is proven to be differentially flat, one can analytically write down a feasible parametrized family of trajectories. Proving flatness results for an n-dof bipedal robot does not guarantee the existence of reasonable walking solutions satisfying the motion constraints. This work demonstrates planning and tracking control of feasible walking trajectories satisfying motion constraints for a higher degree-of-freedom four-link biped with a knee joint in each leg.

Keywords: Robotics, Algebraic methods, Optimization, Under-actuated Biped, Differential flatness.

1. INTRODUCTION

In case of walking robots, under-actuation makes the robot motion more human like since human walking does have phases of under-actuation McGeer (1990); Collins et al. (2001). One approach of planning and controlling trajectories is based on mechanically redesigning the biped, with center of mass at the hip for each leg, such that the continuous component of its overall hybrid dynamics becomes differentially flat Sangwan and Agrawal (2007). Once this class of under-actuated bipeds is proven to be differentially flat, a family of feasible parameterized trajectories can be written down analytically and then can be numerically optimized to satisfy additional motion constraints. Although the previous flatness result was proven for an n-dof bipedal robot with potentially multiple joints in each leg but it was numerically demonstrated with a simple biped with only two links without a knee joint. For bipeds with more joints, it does not directly imply that reasonable walking solutions will always exist because differential flatness only guarantees existence of dynamically feasible periodic trajectories but not necessarily trajectories feasible w.r.t additional motion constraints such as ground clearance, positive ground normal reaction etc. The key contribution in this note is, planning and tracking control of feasible walking trajectories based on the property of differential flatness combined with numerical optimization for a higher degree-of-freedom (DoF) biped having four links with a knee joint in each leg. A full-state feedback controller based on the differentially flat structure of the dynamics is also shown to track the planned trajectories in presence of initial errors.

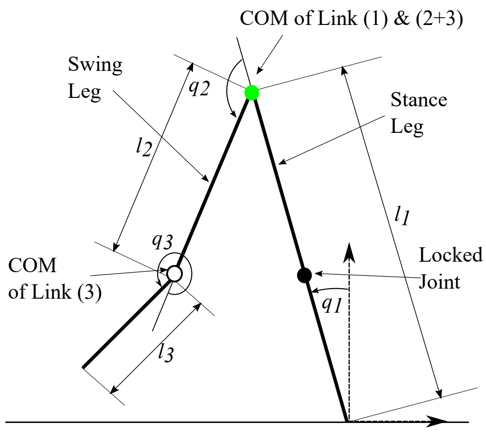

Fig. 1. A four-link planar bipedal robot with a knee joint in each leg.

2. FOUR-LINK BIPED

In this section, the flatness based design methodology is applied to a more complicated biped with knees shown in Fig. 1. The knee and hip joints are actuated whereas the ankle joints are unactuated. Besides actuators, knee joints also have a solenoid actuated latch that is used to lock these joints. At any given instant one of the legs (stance leg) is in contact with ground and the other leg (swing leg) is swinging freely in air. Although the biped has four joints but the knee joint of the stance leg is always locked and hence at any given instant the biped has a maximum of three degrees-of-freedom. The swing and stance legs interchange roles instantaneously when the swing leg hits the ground at ground impact as shown in 2. Following the design methodology presented in Sangwan and Agrawal

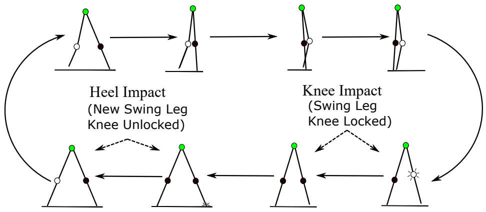

Fig. 2. A four-link planar bipedal robot with a knee joint in each leg.

(2007), COM of both legs is at the hip joint. This COM placement is achieved by first placing the COM of shank segment at the knee joint followed by the placement of combined COM of shank and thigh at the hip joint by means of counter-masses. A complete dynamic model of the biped consists of four separate models (i) 3-DoF phase (ii) 2-DoF phase (iii) the impact model for heel impact and (iv) impact model for knee impact. All of these are derived using energy method.

3. DIFFERENTIAL FLATNESS BASED TRAJECTORY PLANNING

The dynamics for both continuous phases of this system is such that flat outputs with total relative degree equal to the number of states exist. There also exists difeomorphism between state space and flat output space. Outputs for 3-DoF phase are as follows

$$y_1 = q_1 + \bar{m}_{22}q_2 + \bar{m}_{33}q_3, \qquad y_2 = \bar{m}_{33}(q_1 + q_2 + q_3). \quad (1)$$

Parameters $\bar{m}_{22}$ and $\bar{m}_{22}$ are non-dimensionalized inertia parameters appearing in the dynamic model. y_1 with relative degree four and y_2 with relative degree two makes the total relative degree of outputs equal to six i.e. equal to number of states. Similarly, for the 2-DoF phase, flat output is given by:

$$y_1 = q_1 + \bar{m}_{22}q_2, \quad (2)$$

with relative degree four. To satisfy the dynamic feasibility and periodicity requirements, flatness framework is used to generate a family of cyclic dynamically feasible trajectories having a set of free parameters. Then an SQP based optimization routine is used to modulate those free parameters such that the motion constraints mentioned previously are also satisfied. The planning has to be done for two distinct phases the 3-DOF phase from heel impact to knee impact and the 2-DOF phase from knee impact to next heel impact as shown in Fig.2. Critical instances like pre-heel and knee impact states, and post-heel and knee impact states, were made the anchor points during the numerical optimization. Then output's collocation function of time were chosen such that one part of the collocation function is used to satisfy the anchor states and other is used in modulating the trajectories to satisfy the constraints.

4. RESULTS AND CONCLUSIONS

Feasible walking trajectories shown in Fig. 3 are obtained using SQP based numerical optimization over a family of dynamically feasible periodic trajectories, with motion constraints such as positive ground normal reaction, positive heal height. Solid blue lines in this figure are planned

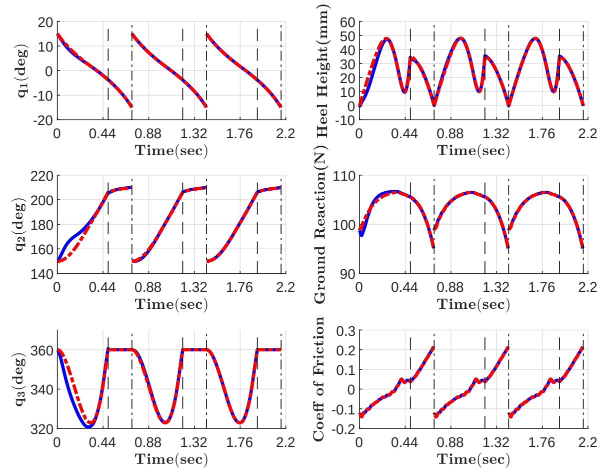

Fig. 3. [Left] Joint angle trajectories, [Right] various constraints imposed during optimization, (dash line - planned trajectory, solid line - actual trajectory).

trajectories and dashed lines are trajectories of the system with the full state feedback controller having some initial errors. Clearly, trajectories converge to the planned even in presence of initial errors and joint angles stay within reasonable range. Also Heel height and normal reaction stay positive and coefficient of friction is within acceptable range.

In conclusion, this work demonstrated construction of feasible walking trajectories based on the property of differential flatness for a biped with higher degrees-of-freedom with a knee joint in each leg. These trajectories were constructed using numerical optimization over a family of dynamically feasible trajectories.The optimizer only had to satisfy the motion constraints such as positive ground normal reaction and ground clearance due to guaranteed periodicity because of flatness and collocation function. Simulations with a linear full-state feedback controller is also shown to eliminate initial errors in the trajectories.

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