Towards Model Predictive Control for Maintaining a Hard Infection Cap during an Outbreak of Dengue Fever

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Abstract: A dynamical model for an outbreak of dengue fever with countermeasures as control inputs is considered. We formulate an optimal control problem (OCP) for maintaining a hard infection cap with preferably low control effort and study different scenarios numerically. To this end, we solve the OCP in both open and closed loop using model predictive control (MPC).

Keywords: epidemiology, hard infection cap, dynamic models, optimal control, model predictive control, numerical optimisation

1. INTRODUCTION

Mathematical models have been used to model epidemics since decades. For instance, compartmental models, where people are characterised by their state of infection, are used to describe the spread of the disease, see, e.g., Hethcote (2000). Due to the ongoing COVID-19 pandemic researchers proposed a variety of compartmental models tailored to model particular characteristics of the disease, see e.g. Grundel et al. (2022). Here, optimal control problems (OCPs) are formulated with countermeasures as control inputs and solved to determine a reasonable (theoretically optimal) strategy. Here, the main goal is to maintain a hard infection cap while keeping drawbacks resulting from the enforced countermeasures as low as possible. However, besides COVID-19, a couple of vector-borne diseases seriously endangering public health are re-emerging in Europe. Hence, we aim at transferring recently proposed methods, see e.g. Grundel et al. (2021), to determine (near) optimal intervention strategies to a dengue fever model, see e.g. Fischer et al. (2019). In particular, we propose an MPC scheme to solve the problem and study the impact of the choice of the prediction horizon length and weights in the objective function on the total number of infections.

2. MODEL AND PROBLEM FORMULATION

We consider the model studied in Fischer et al. (2019) consisting of two species: humans and mosquitos (vectors). Transmission may occur if a susceptible mosquito bites an infectious human or if an infectious mosquito bites a susceptible human.

2.1 System dynamics

Let S_h, V_h, I_h, and R_h denote the total number of susceptible, vaccinated, infected, and recovered people and A_m, S_m, and I_m denote the aquatic (larves), susceptible, and infected mosquitos, respectively. Then, the dynamics for the humans and mosquitos are given by

$$\begin{aligned}
\dot{S}_h &= \mu_h N_h + \theta V_h - \left(\frac{B\beta_{mh}}{N_h} I_m + \psi + \mu_h\right) S_h \\
\dot{V}_h &= \psi S_h - \left(\theta + \sigma\frac{B\beta_{mh}}{N_h} I_m + \mu_h\right) V_h \\
\dot{I}_h &= \frac{B\beta_{mh}}{N_h} I_m \left(S_h + \sigma V_h\right) - (\eta_h + \mu_h) I_h \\
\dot{R}_h &= \eta_h I_h - \mu_h R_h \\
\dot{A}_m &= \varphi\left(1 - \frac{A_m}{3N_h}\right)(S_m + I_m) - (\eta_A + \mu_a + c_a) A_m \\
\dot{S}_m &= \eta_a A_m - \left(\frac{B\beta_{hm}}{N_h} I_h + \mu_m + c_m\right) S_m \\
\dot{I}_m &= \frac{B\beta_{hm}}{N_h} I_h S_m - (\mu_m + c_m) I_m.
\end{aligned}$$

The control $u = (\psi, c_a, c_m) \in L^\infty_{\mathrm{loc}}([0,\infty), [0,1]^m)$, $m = 3$, consists of the vaccination rate ψ as well as the rates of larvicide c_a and adulticide c_m, respectively. In the remainder, we collect all states in $x(t) \in \mathbb{R}^n$, $n = 7$, and write $\dot{x}(t) = f(x(t), u(t))$.

2.2 Optimal control problem

Our goal is to maintain a hard infection cap, i.e.,

$$I_h(t) \;\leq\; I_{\max} \qquad \forall\, t \geq 0,$$

with as little control effort as possible. This motivates the following OCP

$$\min_u \quad J(x_0, u) = \int_0^{t_f} \ell(x(t; x_0, u), u(t))\,\mathrm{d}t \tag{1a}$$

$$\text{s.t.} \quad \dot{x}(t) = f(x(t), u(t)), \quad x(0) = x^0 \tag{1b}$$

$$I_h(t) \leq I_{\max} \quad \forall\, t \geq 0 \tag{1c}$$

with stage costs $\ell : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$,

$$\ell(x, u) \;:=\; \omega\left(\frac{I_h}{I_{\max}}\right)^2 + \frac{1-\omega}{m}\,\|u\|_2^2$$

and non-negative weights $\omega \in [0, 1]$.

We enforce the controls to be constant over one week reflecting the fact that it takes time to implement the countermeasures. Moreover, we use model predictive control (MPC) to mimic real-life decision making by updating the control variables when novel data is available. Given step size $\Delta t > 0$, prediction horizon length $N \in \mathbb{N}_{\geq 2}$, and the current time instant $k \in \mathbb{N}_0$, the three main steps of MPC are

(1) measure current state $\hat{x} = x(k\Delta t)$,
(2) solve the OCP (1) on $[k\Delta t, (k + N - 1)\Delta t)$ to get optimal control $u_k : [k\Delta t, (k + N - 1)\Delta t)$,
(3) implement solution $\mu(k) = u_k(k\Delta t)$ and increment $k \leftarrow k + 1$.

3. NUMERICAL RESULTS

In our simulations we set $N_h = 100,000$ as well as

$$S_h^0 = 99,990, V_h^0 = 0, I_h^0 = 10, R_h^0 = 0,$$
$$A_m^0 = 300,000, S_m^0 = 300,000, I_m^0 = 0.$$

The values of the system parameters are listed in Table 1. We solve the OCP (1) both in open and closed loop over a time window of one year.

Table 1. Overview of all parameters

symbol	description	value
N_h	total human population	100,000
B	average biting rate	0.8
β_{mh}	infection rate from vector to human	0.375
β_{hm}	infection rate from human to vector	0.375
μ_h^{-1}	average life expectancy of humans	$80 \cdot 365$
η_h^{-1}	average infectious time of humans	3
μ_m^{-1}	average life expectancy of mosquitoes	10
η_a	maturation rate of larvae	0.08
φ	amount of eggs per breeding place	6
μ_a	natural death rate of larvae	0.25
σ	efficacy of the vaccine	0.2
θ	waning immunity	0.05

We study the impact of the different choices of the weighting parameter $\omega \in [0, 1]$ and the prediction horizon length $N \in \mathbb{N}_{\geq 2}$ on the total nunber of infections within one year. Results can be found in Figure 1. Note that in all scenarios the hard infection cap is maintained. If we do not penalise the number of infections ($\omega = 0$) the infection cap is reached and the outbreak evolves faster. The higher the weight on penalising infections, the more people are vaccinated (top to bottom). Moreover, the total number of infections for different combinations of ω and N is listed in Table 2. Both, in open and closed loop, increasing

Table 2. Total number of infections within one year depending on weighting ω and horizon N.

N \ ω	0	0.001	0.005	0.01	0.5
2	95422	91194	87334	77446	10036
4	95359	89693	58552	41937	4389
6	95313	87914	43004	29524	2959
8	95382	87769	36919	24284	2311
12	95318	89256	35951	20859	1681
open loop	95296	89715	76131	50060	1552
$\|u\|_\infty$	0.0557	0.0626	0.0738	0.0771	0.1116

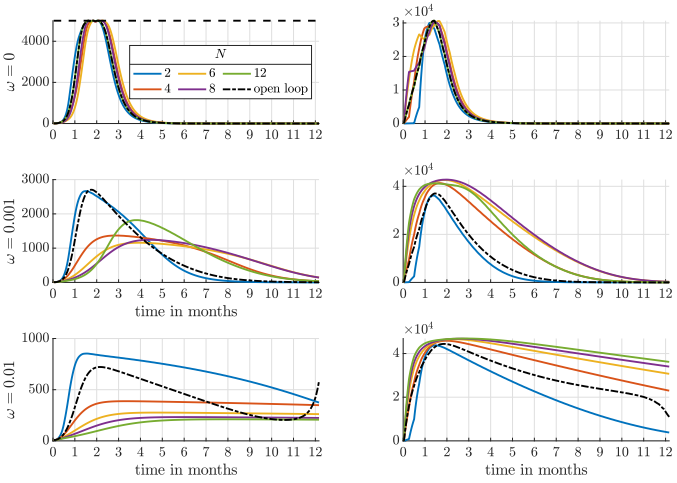

Fig. 1. Results with cap $I_{\max} = 5,000$ for increasing weight ω from top to bottom: I_h (left), V_h (right).

the weight ω yields a reduction of the total number of infections for a fixed prediction horizon length N (left to right).

For fixed weight ω, we observe that in most cases, increasing the horizon length N also results in lower case numbers. However, there is an exception for $\omega = 0.001$: If we increase the prediction horizon from $N = 8$ to $N = 12$, the total number of infections slightly increases. However, the weight $\omega = 0.001$ for penalising the number of infections is very small. Thus, it is cheaper to accept more infections for the trade-off of less control effort. This becomes particularly prominent in open loop with $\omega = 0.01$. In Figure 1 (bottom left), the infection numbers rise towards the end of the prediction horizon since we cannot reduce the control effortwithout causing the number of infections to explode within the considered time window.

4. CONCLUSIONS AND OUTLOOK

We studied an OCP for maintaining a hard infection cap in case of an outbreak of dengue fever. We found that the choice of both the weights in the objective function and the prediction horizon length are crucial to reduce the total number of infections. Future research will consider several serotypes to model vaccination more accurately.

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